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MATHEMATICAL MODEL OF HYDRODYNAMICS OF A GLASS-MELTING TANK

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The importance of studying the hydrodynamics of the glass-melting tank is substantiated. The main equations of a numerical model of the tank hydrodynamics are given. The calculation results of temperature fields and streamlines are given for a glass-melting furnace with an output of 300 tons per day. The adequacy of the calculation results as applied to real furnace operating conditions is demonstrated. The developed model of hydrodynamics of the melting tank can be used in solving applied problems of designing glass-melting furnaces.

The evolution of glass-melting furnaces indicates that the specific glass melt output is the most critical of numerous parameters characterizing the engineering efficiency of the furnace. This parameter determines not only the total amount of glass produced during the furnace campaign and the thermal efficiency of the process, but the furnace cost as well. The specific output of furnaces to a large extent depends on the melting temperature. The industry considers increasing the melting temperature, which is understood as the maximum instrumentally monitored temperature of the inner roof surface, as one of the main methods for improving the furnace efficiency. At a certain phase of the evolution of glass-melting furnaces, an increase in melting temperature from 1500 to 1580°C contributed to raising the glass melt output up to 2 – 2.5 tons/m^2 per day. However, this is actually the limiting temperature level for the domestic dinas refractory (GOST 3910–75) used for the furnace roof. Using high-quality imported dinas materials makes it possible to increase the roof temperature up to 1600°C, and electromelted baddeleyite-corundum refractories — to 1640 – 1650°C [1]. However, in general the possibility of intensifying the furnace performance by raising its melting temperature is limited by the resistance of refractory materials and is now virtually exhausted.

Despite the obvious dependence of specific output on melting temperature, the latter is a significant, but not the only factor determining the furnace efficiency. In our opinion, increasing the specific output of a furnace is an integrated challenge involving a number of technological and design problems. A significant factor is the design of the melting tank, where the optimization of its geometrical parameters involves a detailed study of the hydrodynamic regularities of the melt.

Let us represent the physical model of a glass-melting furnace as two zones divided by the glass melt surface and separated from the ambient medium by the enclosing refractory brickwork. In this scheme the technological process zone (TPZ, the melting tank) is the main one and the heat generation zone (HGZ, the flame space) is the auxiliary zone providing for certain conditions in the TPZ. The most important prerequisites to obtaining high-quality glass under a preset furnace output include the transfer of a required amount of thermal energy into the technological process zone and ensuring the stay of the melt in the tank for a time required for all glass-melting reactions to be completed. The first prerequisite is due to the endothermic type of glass-formation reactions and the second one is related to the sequence of the glass-melting stages and their time dependence. The process of heat transfer from the HGZ to the TPZ is the predominant one for flame and gas-electric glass-melting furnaces with respect to the energy dynamics of the melting tank. The efficiency and the duration of the multistage technological process depend both on the amount of thermal energy assimilated by the glass melt and on its distribution in the TPZ volume. While the first component to a large extent depends on the efficiency of external heat transfer, the second one depends on thermophysical properties of the glass batch and glass melt and on the hydrodynamics of the melting tank.

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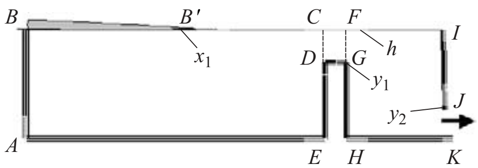

Fig. 1. Scheme of glass melt migration in the longitudinal section of the melting tank of a glass-melting furnace.

The melting tank hydrodynamic in this case is understood as the set of regularities of heat and mass exchange in the tank forming the volumetric temperature field of the glass melt. The influence of the external heat transfer parameters on the heat and mass transfer within the glass melt volume are manifested by the temperature profile on the glass melt surface. The nature of glass melt flows in the tank depends on several factors. First, it depends on the inhomogeneity of the temperature field on the glass melt surface, which generates glass melt convection in accordance with thermal gravitation convection criteria. Second, the formation of circulation contours is initiated by the design elements of the tank impeding natural migration of the melt. Finally, the glass melt migration is largely determined by the obvious temperature dependence of its viscosity, since the tangential stress in the medium forming stream turbulence is proportional to the dynamic viscosity coefficient. Thus, we recognize a close relationship between the heat and mass transfer in the tank and the exterior heat exchange in the flame space, as well as the possibility of analyzing them separately. With such an approach, the results of solving the exterior problem serve as initial data for the interior problem. This separation principle has been used in the development of a mathematical model for a glass-melting furnace [2, 3].

The intensification of heat and mass transfer processes in the melting tank achieved by different methods enables one to solve other applied problems related to the increasing the specific output of the furnace. The point is, primarily, extending the time of stay of "new" glass melt portions in the tank, in particular, by increasing the multiplicity of glass melt circulation in the main convection cycles (charging and working cycles). This not only decreases the probability of substandard glass melt arriving at the working zone, but also improves its homogeneity. The increase in the multiplicity of the charging convection cycle is especially significant, as this has a direct impact on the intensity of the initial melting of the batch. The setting of such problems of fluid dynamics of the melting tank has become possible due to computerized numerical modeling of technological processes. At the same time it should be noted that the applied significance of modeling results largely depends on correct formalizing of fluid dynamic processes.

Thermoconvection phenomena in fluids in the general case are described by a system of nonlinear partial derivative equations. This system includes the equations of the laws of conservation of energy, impulse, and mass of material and the rheological equation of the state of the medium [4 – 7]. The difficulty of analytically solving fluid dynamics problems implies the use of various simplifying assumptions. The most typical assumption is implied in the problem setting by specifying conditions in which the forced motion of the medium is absent and the free motion domain is bounded by two parallel walls (top and bottom). Furthermore, the physical characteristics of the medium are assumed to be constant, its density depending only on temperature (and this dependence is taken into account only in the gravity expression), the surface temperatures are constant, etc. It is understandable that the practical applicability of results obtained with such assumptions seems rather conventional. Applied problems of designing glass-melting furnaces require a more detailed setting of the problem of the glass melt heat convection necessarily taking into account the exterior (forced) motion of the melt through the tank neck.

Although the temperature field of the glass melt is volumetric, at the first stage of the study we can limit ourselves to the analysis of the temperature, pressure, and velocity fields in the two-dimensional setting (i.e., along the longitudinal section of the tank). The validity of such simplification is due to the fact that the width of the tank is 3 – 10 larger than its depth. Analyzing a stationary (steady) process typical of continuous furnaces, it is advisable to mathematically describe the convection phenomena in the melting tank using the "vorticity – stream function" variables. Using the equations from [2] the problem for the medium with variable thermophysical properties migrating in the area of glass melt motion shown in Fig. 1 can be described by the following system of equations in dimensionless form:

vorticity transfer equation:

$$\frac{\partial}{\partial X}\left(\omega\frac{\partial\psi}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(\omega\frac{\partial\psi}{\partial X}\right)=$$
$$\frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial X}\left(\mu_{\mathrm{ef}}\frac{\partial\omega}{\partial X}\right)-\frac{\partial}{\partial Y}\left(\mu_{\mathrm{ef}}\frac{\partial\omega}{\partial Y}\right)\right]-\frac{1}{\mathrm{Fr}}\frac{\partial\rho}{\partial X}+R_{\omega}; \tag{1}$$

stream function equation:

$$\frac{\partial}{\partial X}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial Y}\right)=\omega; \tag{2}$$

equations specifying vorticity and stream functions:

$$\omega=\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}; \tag{3}$$

$$\left.\begin{aligned}\rho U&=\frac{\partial\psi}{\partial Y};\\ \rho V&=-\frac{\partial\psi}{\partial X};\end{aligned}\right\} \tag{4}$$

energy equation:

$$c_p\left[\frac{\partial}{\partial X}\left(T\frac{\partial\psi}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(T\frac{\partial\psi}{\partial X}\right)\right]=\frac{1}{\mathrm{Pe}}\left[\frac{\partial}{\partial X}\left(\lambda_{\mathrm{ef}}\frac{\partial T}{\partial X}\right)-\frac{\partial}{\partial Y}\left(\lambda_{\mathrm{ef}}\frac{\partial T}{\partial Y}\right)\right],\qquad(5)$$

where X and Y are the longitudinal and vertical coordinates expressed in fractions of the tank neck height y_2 (Fig. 1); U and V are the longitudinal and vertical components of the glass melt velocity expressed in the fractions of its average velocity in the tank neck V_0; ρ, μ_{ef}, and λ_{ef} are the dimensionless values of density, effective viscosity, and thermal conductivity of the glass melt; T and c_p are the dimensionless temperature and specific heat capacity of the glass melt; ψ and ω are the dimensionless values of the stream function and flow vorticity; their physical meaning ensues from expressions (3) and (4); the dimensionless values of thermophysical parameters and glass melt temperature are expressed in fractions of the values of these parameters corresponding to a certain basic temperature T_0; Re, Fr, and Pe are the Reynolds, Froude, and Peclet similarity numbers; the source term R_ω is determined by the change in the kinetic energy of the averaged glass melt flow and its effective viscosity.

As the area of the glass melt motion has a rather complicated contour, we will use a nonuniform grid (Fig. 2). Let us apply the control volume method [8, 9]. Having approximated the differential equations on this grid according to the recommendations given in the cited studies, instead of the Poisson stream function equation (2) we obtain

$$B_P\psi_P = B_E\psi_E + B_W\psi_W + B_N\psi_N + B_S\psi_S + D_{\psi,P}\,\omega_P,\qquad(6)$$

where the indexes P, E, W, N, and S at the values of the stream function ψ and vorticity ω represent their values in the respective nodes of the finite-difference grid (Fig. 2); the thermal conductivity values B with indexes P, E, W, N, and S include the grid steps along the coordinates X and Y, as well as the dimensionless density of the glass melt at the points e, w, n, and s; the coefficient $D_{\psi,P}$ is equal to the cross-sectional area of the control volume indicated in Fig. 2 by shading.

The vorticity transfer equation (1) is transformed according to the recommendation from [8, 10] using the counterflow scheme and S. Patankar's corrective [10]. As a result we obtain a discrete analog of the vorticity equation in the following form:

$$\omega_P = C_{\omega,E}\,\omega_E + C_{\omega,W}\,\omega_W + C_{\omega,N}\,\omega_N + C_{\omega,S}\,\omega_S + D_\omega.\qquad(7)$$

In this case the vorticity conductivity C_ω at the nodes P, E, W, N, and S in addition to the grid steps include the convective transfer coefficients proportional to the Reynolds number and the diffusion transfer coefficients proportional to the viscosity μ_{ef}.

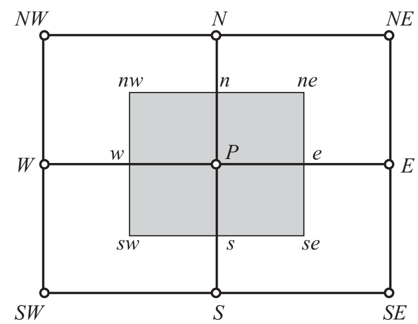

Fig. 2. Pattern of finite-difference grid.

The glass melt energy equation (5) differs from the vorticity transfer equation only by the presence of a factor at the convective terms (specific heat capacity c_p) and the absence of the source term. Therefore, the complete finite-difference temperature equation can be represented in the following form:

$$T_P = C_{T,E}T_E + C_{T,W}T_W + C_{T,N}T_N + C_{T,S}T_S,\qquad(8)$$

where the meaning of the thermal energy conductivities C_T is analogous to C_ω in Eq. (7).

To close equation systems (6) – (8), the discrete analog of each differential equation in (1) – (5) has to be supplemented by algebraic relations approximating the boundary conditions at boundary points. The solution of this problem requires an explicit representation of the geometry of the glass melt motion area, although without yet specifying particular sizes.

According to the scheme represented in Fig. 1, the boundary conditions on the stream function ψ are strictly defined on the *BAEDGHK* and *IJ* contours and in the *JK* channel. In the *BAEDGHK* contour $\psi = 0$, whereas the stream function distribution in the section *JK* is determined by the glass melt velocity profile. Assuming the glass melt flow directed to the working zone to be laminar, we write

$$U(L, Y) = 6Y(1 - Y),\qquad(9)$$

where $Y = y/y_2$; $L = l/y_2$ is the dimensionless melting tank length (l is the tank length).

Using the first of relations (4) we obtain

$$\psi(L, Y) = \int_0^Y U(L,\eta)\mathrm{d}\eta = 6\left(\frac{1}{2}Y^2 - \frac{1}{3}Y^3\right) = 3Y^2 - 2Y^3,$$

where η is the integrating variable.

Consequently, $\psi = 1.0$ on the line *IJ*. The same value of the stream function is preserved on the *B′CFI* line as well.

According to the mass conservation law, the quantity of glass melt arriving at the melting tank on the line *BB′* should be equal to the quantity of glass melt removed at the working zone, i.e., the stream function ψ should vary here from 0

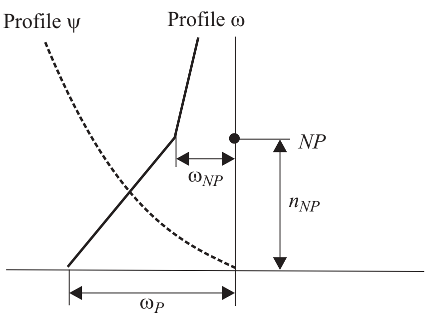

Fig. 3. Scheme of boundary parameters.

to 1.0. The regularity of this variation is determined by the kinetics of the physicochemical processes of the formation and melting of glass. In the first approximation it can be assumed that the specified processes ensure the required glass melt consumption under its parabolic velocity profile, as in Eq. (9),

$$V(H, X) = -\frac{6}{X_1}\left[\frac{X}{X_1} - \left(\frac{X}{X_1}\right)^2\right], \qquad (10)$$

which leads to the distribution of ψ:

$$\psi(H, X) = \left(\frac{X}{X_1}\right)^2\left(3 - 2\frac{X}{X_1}\right),$$

where $X_1 = x_1/y_2$ (x_1 is the length of the glass melt area covered by the batch, see Fig. 1); $H = h/y_2$ is the dimensionless height of the glass melt layer (h is the glass melt depth in the tank, see Fig. 1).

The boundary conditions on vorticity ω will be specified in accordance with D. B. Spalding's recommendations [8]. According to these recommendations, the stream function ψ near the wall changes according to a cubic law, whereas the stream vorticity ω changes according to a linear law, which satisfies the Woods condition [9]:

$$\omega_P = \left[\frac{3(\psi_{NP} - \psi_P)}{\rho n_{NP}^2} - \frac{\omega_{NP}}{2}\right],$$

where the indices and the variables correspond to the scheme represented in Fig. 3.

The boundary conditions on vorticity ω on the glass melt surface can be represented as follows:

$$\omega = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}.$$

On the line BB' we have $U = 0$ and, accordingly, $\omega = -\partial V/\partial X$. In accordance with Eq. (10) we can write

$$\omega(H, X) = \frac{6}{X_1^2}\left(1 - 2\frac{X}{X_1}\right).$$

Similarly, $V = 0$ in the section JK and $\omega = \partial U/\partial Y$, i.e.,

$$\omega(L, Y) = 6(1 - 2Y).$$

On the line $B'CFI$ we also have $V = 0$ and $\omega = \partial U/\partial Y$. Considering that this line corresponds to the free surface on which the adhesion condition is absent and, consequently, there is no vorticity, it is possible in this case to set the zero boundary condition.

Let us consider the temperature boundary conditions. The temperature on the glass melt surface is known from the solution to the exterior problem, and the heat flux densities are known as well. Before solving the problem of the tank hydrodynamics, it is necessary just to approximate the zonal temperature distribution by a continuous function. This operation can be performed in any statistical software format.

On the boundaries AB, AE, IJ and HK the boundary condition of the III kind are specified:

$$-\lambda_{\text{ef}}\frac{\partial T}{\partial n} = k(T - T_{\text{amb}}),$$

where n is the dimensionless normal; k is the dimensionless heat transfer coefficient; T_{amb} is the dimensionless temperature of the ambient medium.

Having approximated the derivative in this expression on a 2nd order nonuniform grid, we obtain the discrete analogs of the boundary conditions on the solid boundaries:

the left wall (W is the boundary point; $\partial T/\partial n = -\partial T/\partial x$)

$$T_W = [(X_E - X_W)^2 T_E - (X_P - X_W)^2 T_P + k(X_E - X_P)(X_P - X_W)(X_E - X_W) T_{\text{amb}}]/[(X_E - X_W)^2 - (X_P - X_W)^2 + k(X_E - X_P)(X_P - X_W)(X_E - X_W)];$$

the right wall (E is the boundary point; $\partial T/\partial n = \partial T/\partial x$)

$$T_E = [(X_E - X_P)^2 T_P - (X_E - X_W)^2 T_P - k(X_P - X_W)(X_E - X_P)(X_E - X_W)]/[(X_E - X_P)^2 - (X_E - X_W)^2 - k(X_P - X_W)(X_E - X_P)(X_E - X_W)];$$

the melting tank bottom (S is the boundary point, $\partial T/\partial n = -\partial T/\partial y$)

$$T_S = [(Y_N - Y_S)^2 T_P - (Y_P - Y_S)^2 T_N + k(Y_P - Y_S)(Y_N - Y_P)(Y_N - Y_S)]/[(Y_N - Y_S)^2 - (Y_P - Y_S)^2 + k(Y_P - Y_S)(Y_N - Y_P)(Y_N - Y_S)].$$

One can set adiabatic boundaries as the boundary conditions on the boundaries ED, DG, GH, and JK. In particular, when approaching the spillway from the right we obtain

$$T_E = \frac{(X_E - X_W)^2 T_P - (X_E - X_P)^2 T_W}{(X_E - X_W)^2 - (X_E - X_P)^2}.$$

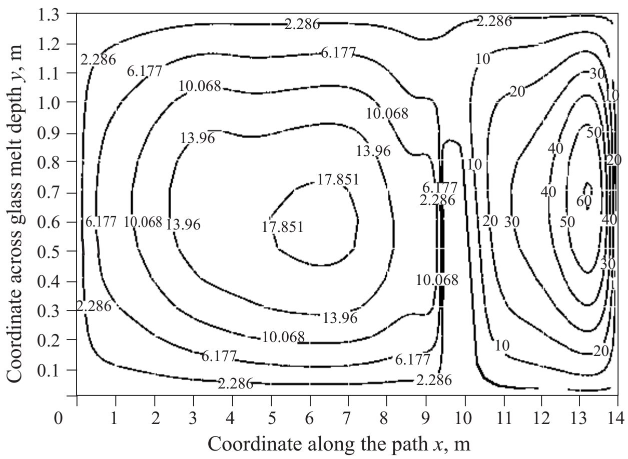

Fig. 4. Glass melt flow in the melting tank (numbers on the curves represent normalized stream function).

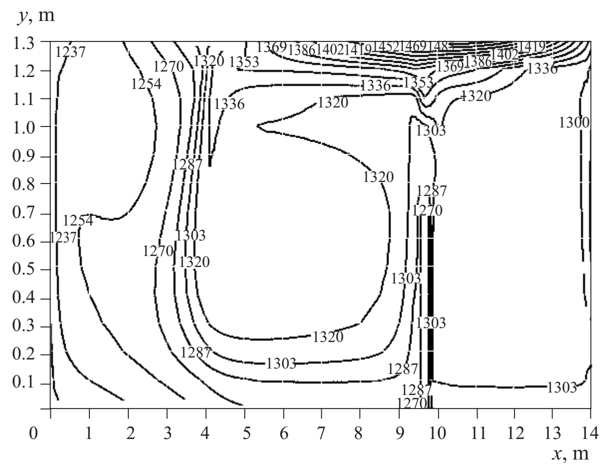

Fig. 5. Temperature field of glass melt in the melting tank of the glass-melting furnace (numbers on the curves denotes glass melt temperature, °C).

The same expression is true for the section JK. When approaching from the left we have

$$T_W = \frac{(X_E - X_W)^2 T_P - (X_P - X_W)^2 T_E}{(X_E - X_W)^2 - (X_P - X_W)^2}.$$

Finally, for the spillway top (line DG) we have

$$T_S = \frac{(Y_N - Y_S)^2 T_P - (Y_P - Y_S)^2 T_N}{(Y_N - Y_S)^2 - (Y_P - Y_S)^2}.$$

Having the complete information on the finite-difference setting of the problem, it is possible to select a method for solving the discrete analog equations. Due to the substantial nonlinearity of the specified equations (all thermophysical parameters vary significantly with increasing temperature) the only way to solve them is to use the iterative coordinate displacement methods. For the same reason the most convenient of the numerous available methods is the consecutive displacement method or the Gauss – Zeidel method [11, 12]. It is possible to apply various successive over- and under-relaxation methods. However, on modern fast personal computers this does not give significant time savings.

The model of the melting tank fluid dynamics has been adapted for a container-glass furnace with an output of 300 tons per day. The length, width, and depth of the melting tank are 13.62, 8.5, and 1.3 m, respectively. The width and the height of the tank neck are equal to 0.8 and 0.3 m, and the height and the width of the barrier spillway are 0.8 and 0.4 m, respectively. Other source data are specified in [2]. The temperature distribution on the glass melt surface was taken from the solution to the exterior heat transfer problem. The maximum temperature zone is located above the spillway installed in the quellpunkt zone.

To illustrate the calculation results, let us consider the distribution of glass melt streamlines (normalized by the mass flow rate of the glass melt in the tank neck) and temperatures in the longitudinal section of the melting tank. The data in Figs. 4 and 5 corroborate the quantitative and qualitative adequacy of the obtained results to the generally accepted concepts of the performance of a glass-melting tank. The temperature distribution on the glass melt surface and the presence of a spillway contribute to the formation of two clearly defined convection contours, the dividing line between them corresponding to the classical definition of the "quellpunkt" notion.

The intense circulation of the melt is largely due to the efficient thermal insulation of the tank brickwork [3] and the high specific glass melt output (about 2.6 tons/m^2 per day). In comparing the stream function determined by the rate of removal of the glass melt to the working zone and a stream function equal to 1.0 (in the dimensionless expression), the maximum value of the stream function in the tank is 60.66, i.e., the circulation multiplicity is more than 60. The intense glass melt circulation contributes to homogenizing the temperature in the tank volume. As a consequence, high technologically suitable temperatures are achieved in the bottom glass melt layers and in the tank neck.

Thus, the developed numerical model of hydrodynamics of a melting tank combined with the previously described model of exterior heat transfer [3] can be used for solving applied problems in designing or evaluating existing designs of glass-melting furnaces.

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